Martingale transforms and Fourier multipliers

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D. L. Burkholder and G. Wang established L^p estimates for martingales with comparable quadratic variation. The result has important applications in the study of L^p operator norm of singular integral operators and Fourier multipliers. This is so because many such operators may be represented as martingale transforms of martingales obtained by composing space-time Markov process with the corresponding parabolic functions. I will discuss estimates obtained over the years by this method, in particular joint results obtained with Rodrigo Bañuelos by using stochastic calculus of discontinuous processes. For instance, the Fourier multiplier with the symbol

$$M(\xi) = \frac{\frac{1}{2}\int_{\mathcal{S}}(\xi,\theta)^2\varphi(\theta)\mu(d\theta) + \int_{\mathbf{R}^d}[1-\cos(\xi,z)]\phi(z)V(dz)}{\frac{1}{2}\int_{\mathcal{S}}(\xi,\theta)^2\mu(d\theta) + \int_{\mathbf{R}^d}[1-\cos(\xi,z)]V(dz)},$$

has norm at most $p^*-1 = \max\{p-1, 1/(p-1)\}$ on $L^p(\mathbf{R}^d)$ for $1 < p < \infty$. Here $(\ ,\ )$ is the usual scalar product, V is a Lévy measure on $\mathbf{R}^d$,

$$\int_{\mathbf{R}^d} \min(|z|^2, 1)V(dz) < \infty,$$

$\mu \geqslant 0$ is a finite Borel measure on the unit sphere $\mathcal{S} \subset \mathbf{R}^d$, ϕ and φ are complex-valued functions, and $\|\phi\|_\infty \leqslant 1$, $\|\varphi\|_\infty \leqslant 1$.

References

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[2] G. Wang, *Differential subordination and strong differential subordination for continuous-time martingales and related sharp inequalities*, Ann. Probab. 23 (2) (1995) 522–551.