Harmonic Mappings and the Nitsche Conjecture

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The Nitsche conjecture is deeply rooted in the theory of doubly connected minimal surfaces. However, it is commonly formulated as a question of existence of a harmonic homeomorphism between circular annuli

$$h\colon \mathbb{A} = A(r,R) \xrightarrow{\text{onto}} A(r_*,R_*) = \mathbb{A}^*$$

In the early 1960s, while attempting to describe all doubly connected minimal graphs over a given annulus $\mathbb{A}^*$, J.C.C. Nitsche observed that their conformal modulus cannot be too large. Then he conjectured, in terms of isothermal coordinates, even more;

A harmonic homeomorphism $h\colon \mathbb{A} \xrightarrow{\text{onto}} \mathbb{A}^*$ *exists if an only if*:

$$\frac{R_*}{r_*} \geqslant \frac{1}{2}\left(\frac{R}{r} + \frac{r}{R}\right).$$

This fascinating and engaging problem remained open for a half of a century. In this lecture I will present recent joint work with Leonid Kovalev and Jani Onninen in which we give, among further generalizations, an affirmative answer to his conjecture.