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On continuous unimodular functions on the circle

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The starting point is a beautiful formula of Brézis for the topological degree of a $H^{1/2}$ mapping of the circle into itself by means of its Fourier coefficients. Since the Sobolev space $H^{1/2}$ is in VMO (vanishing mean oscillation), the topological degree, or winding number, is defined by the theory of Brézis and Nirenberg. The topological degree is given by a series which converges absolutely when and only when the function belongs to $H^{1/2}$. When this is not the case, is it possible to obtain the topological degree by means of a convenient process of summation of the series ? This will be discussed using the contributions of Korevaar, Kahane and Bourgain–Kozma. It appears that the $R1$ process of summation ($R2$ is the classical Riemann process)plays a crucial role, together with the Zygmund class small $\lambda^3_{1/3}$. It is an opportunity to investigate the relations between different processes of summation, a subject studied by Marcinkiewicz. The end of the talk is motivated by a question of Brézis and the answers given by Bourgain and myself. The Fourier coefficients of continuous or VMO functions of constant absolute value have interesting properties: if they are summable with a convenient weight for positive indexes, the same is true for all indexes ; the statements are very simple, and the proofs difficult.

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